Student's name

Instructor's name

Course

Date

Homework solutions

Solution 1

The five sets are defined as A = {1, 3, 5, 6, 7, 8, 9, 0}, B = {x | x ~Z j-4 ≤ x ≤ 6}, C = {2, 4, 6, 8, 10, 12, 14}, D = Ø, U = Z.

A) To find the result of A ∪ B ∪ C, it is necessary to perform the following operations:

1) The set B should be written element by element:

B = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6};

2) The union of two sets A and B is performed as follows:

A ∪ B = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9};

3) The union operation of three sets A, B, and C requires an inclusion of all the elements of these sets in a large one:

A ∪ B ∪ C = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14}.

B) To determine the result of A ∩ B ∩ C, it is necessary to perform the two intersection operations in consecutive order:

1) A ∩ B = {0, 1, 3, 5, 6};
2) A ∩ B ∩ C = {6}.

C) To find the difference between the sets A and B, as well as between B and A, perform the two difference operations in consecutive order:

1) A / B = {7, 8, 9};
2) B / A = {-4, -3, -2, -1, 2, 4}.

Solution 2

Given the three sets A, B, and C, prove the next equalities, using binary operations and characteristic function.

A) A \ (A ∩ B) = A \ B

B) (A ∩ B) \ (B \ A) = A

C) A \ (C ∩ B) = (A \ B) ∪ (A \ C)

A) Set equality proofs are usually linear equivalence proofs that use definitions about set operations and logical identities **(**O'Leary 25**).** The following formulas serve as the basis for performing theoretical operations:

1) $X_A(x) = 1 - X_A(x)$

2) $X_{A\cap B}(x) = X_A(x) * X_B(x)$

3) $X_{A\cup B}(x) = X_A(x) + X_B(x) - X_A(x) * X_B(x)$

Therefore, to prove the set qualities, it is necessary to determine the value of the characteristic functions on the both sides of the equation. If characteristic functions on both sides coincide, theoretical correlations are considered to have been proved.

A) The characteristic function of the left side is defined as follows:

$$X_{A*(A\cap B)}(x) = X_A(x) - X_A(x) * X_{A\cap B}(x) = X_A(x) - X_A(x) * X_A(x) X_B(x) = X_A(x) - X_A(x) * X_B(x)$$,

while the characteristic function of the right side is $X_{A/B}(x) = X_A - X_A * X_B$. Therefore, A\ (A ∩ B) = A \ B.

B) Then, the value of the characteristic function on the left side of the equality can be determined consecutively: $X_{(A\cup B)(B/A}(x) = X_{A\cup B}(x) - X_{A\cup B}(x) * X_{B/A}(x)$ that is equal to

$$X_A(x) + X_B(x) - X_A(x) * X_B(x) - (X_A(x) + X_B(x) - X_A(x) * X_B(x)) * (X_B(x) - X_B(x) * X_A(x)) = X_A(x)$$

Hence, the characteristic function proofs the correctness of the correlation:

(A ∪ B) \ (B \ A) = A

C) The left side of the correlation is defined as:

$X_{A(C\cap B}(x) = X_A(x) - X_A(x) * X_{C\cap B}(x) = X_A(x) - X_A(x) * Xc(x) * XB(x)$, while the right side is defined as:

$$X_{(A/B)\cap(A/C)}(x)=X_{A/B}(x)+X_{A/C}(x)-X_{A/B}(x)*X_{A/C}(x)$$

Then, given the characteristic function, the corresponding operations are provided:

$$X_{(A/B)\cap(A/C)}(x)=X_A(x)-X_A(x)*X_B(x)+X_A(x)-X_A(x)*X_C(x)-X_A(x)*X_B(x)*(X_A(x)-X_A(x)*X_C(x))$$

which is equal to:

$$2*X_A(x)-X_A(x)*X_B(x)-X_A(x)*X_C(x)-X_A(x)*X_A(x)+X_A(x)*X_A(x)*X_C(x)=X_A(x)-X_A(x)*X_B(x)*X_C(x)$$

From this perspective, $X_{(A/B)\cup(A/C)}(x)=X_A(x)-X_A(x)*X_B(x)*X_C(x)$

Ultimately, $X_{A(C\cap B}(x)=X_{(A/B)\cup(A/C)}(x)$, showing that A \ (C ∩ B) = (A \ B) ∪ (A \ C).

Based on the calculations provided above, the equalities are proven because the characteristic functions on both sides correlate to one another.

Solution 3

Given the numbers, find the value of the two sets A and B if:

A) A \ B = {a, b}, B \ A = {c, d}, A ∩ B = {x, y, z};

B) A ∪ B = {a, b. c, d, e, f}, A ∩ B = {c, d}, A \ B = {a, e, f};

C) A ∪ B = {a, b, c, d}, A ∩ B = Ø, A \ B = {a}.

A) Since A ∩ B = {x, y, z}, then x, y, and z can be considered elements of both sets at the same time. If A \ B = {a, b} and (A∩ B) ∪ (A\B) = A, then A = {a, b, x, y, z}.

Analogically, since B \ A = {c, d} and A ∩ B = {x, y, z}, then B = {c, d, x, y, z}.

B) Since it is known that (A∩ B) ∪ (A\B) = A, A ∩ B = {c, d}, and A \B = {a, e, f}, then A = {a, c, d, e, f}. Taking into account that B = (A ∪ B)\(A\B), A ∪ B = {a, b. c, d, e, f},, and A \ B = {a, e, f}, then B = {b, c, d}.

C) Since A ∩ B = Ø, then A \B = A = {a}. Correspondingly, B = (A ∪ B) \ A = {b, c, d}.

As a result.

A) A = {a, b, x, y, z}, B = {c, d, x, y, z};

B) A ∪ B = {a, b, c, d, e, f}, B = {b, c, d};

C) A = {a}, B = {b, c, d}.

Solution 4

The power of sets A, B, and C should be found, provided:

A) A = {x ∈ N | x = 2n, n ∈ N};

B) B = {x ∈ N | 1 ≤ x≤ 50};

C) C = {x | x ≥ 5, x ∈ R}.

The power of a set is defined as the set of all subsets that identify its function and indicate its finite or infinite nature (Levin 122).

A) Since $A = \{x \in N \mid x = 2n, n \in N\}$, the set A is infinite. □ $x \in A: x = 2n, n \in N$. All even numbers are unambiguous in the set of natural numbers. Since the equivalence of these sets is apparent, the power of the set of even numbers is equal to the power of the set of natural numbers. Therefore, the set $A = \{x \in N \mid x = 2n, n \in N\}$ has a finite power.

B) The set $B = \{x \in N \mid 1 \leq x \leq 50\}$ contains 50 elements, all of which are natural numbers ranging from 1 to 50. In such a case, the set is finite, and its power is equal to the number of elements included. Therefore, $|B| = 50$. The ultimate conclusion is that the set $B = \{x \in N \mid 1 \leq x \leq 50\}$ has a finite power.

C) All the elements of the set $C = \{x \mid x \geq 5, x \in R\}$ are real numbers that are not smaller than the value 5. Since the set is a half-interval of the real axis, it's power is equal to the values (0,1). Thus, the power of the set $C = \{x \mid x \geq 5, x \in R\}$ is continuum.

Works Cited

Levin, Oscar. *Discrete Mathematics*: *an Open Introduction*. 12th Media Services, 2016.

O'Leary, Michael L. *A First Course in Mathematical Logic and Set Theory*. Wiley, 2016.